

Teacher's Preparatory Guide

Uncertainty Measurements and the Wavelength of Light

Overview:

The ability to accurately measure very small distances is of the utmost importance in nanotechnology. Nanoscale science and engineering occurs between 1 and 100 nanometers but may range up to several hundred nanometers. At this scale, it becomes critical to have accurate measurements. In this lab, students will calculate the wavelength of a laser (on the nanoscale) using simple classroom materials. Students will also take into account uncertainties in their measurements. By incorporating these uncertainties, students will determine a range of acceptable wavelengths of the laser and compare their value to that provided by the manufacturer.

Purpose:

This lab is designed to help students understand how very small measurements can be made (even at the nanoscale) and how the accuracy of those measurements affects their results.

Time Required:

The time required for this lab is one 55-minute period. Part of another period following this should be used for discussion and follow-up. Prior to this activity, students should have been exposed to 1) the concept of measurement uncertainty and 2) the propagation of uncertainties in performing calculations with those measurements.

Level:

High School Physics

Big Idea: Size and Scale; Tools and Instrumentation

Teacher Background:

The ability to make accurate measurements is vital in the scientific world. At the nanoscale, measurements occur at 10^{-9} to 10^{-7} meters and require great accuracy and precision. Understanding uncertainty becomes critical when making measurements at the nanoscale. To further explore measurement at the nanoscale see NNIN's lesson "Is measuring an art or science?" at: <http://www.nnin.org/education-training/k-12-teachers/nanotechnology-curriculum-materials/measuring-art-or-science>.

To understand the limits of these measurements is critical as well. In reporting the results of an experiment where measurements are involved, it is important for the person conducting the experiment to state not only the result, but also the range of uncertainty associated with that result. This gives the reader an idea of a range of an acceptable value. For example, suppose a scientist analyzed some data and calculated the gravity on Mars to be 3.710 m/s^2 . This value by

itself is not sufficient as it excludes the uncertainty. A more valuable result might be $3.710 \pm 0.005 \text{ m/s}^2$. This provides the reader with more information as it indicates the accuracy and precision of the measurement. This informs the reader that the researcher's results show the gravity on Mars is anywhere from 3.705 m/s^2 to 3.715 m/s^2 .

Any measurement tool has some uncertainty intrinsic to it. The accuracy at which the length of a metal rod, for example, can be measured depends on the smallest increment on the measuring device. In a classroom setting, the most common method of performing such a measurement would be to use a meter stick, which is typically graduated to the millimeter. In a typical lab class, most students would record a measurement to the nearest centimeter, and some would record to the nearest millimeter. Both of these recordings, however, do not correctly indicate the accuracy of the meter stick. A correct measurement would show a value pertaining to the smallest increment on the device plus one more digit. That is, for the case with a meter stick, a correct measurement would include a value four digits after the decimal point. The fourth digit represents a tenth of a millimeter and is an estimate. This estimation is where inherent error is found in a measurement. This error is expressed in a measurement as an uncertainty.

There are different options for determining the uncertainty associated with a specific device (e.g., a meter stick, digital scale, graduated cylinder, etc.)¹⁻². For example, the Appalachian State Physics department¹ suggests using a 'one-fifths rule' for uncertainty measurements when using a non-digital device. In this case, the uncertainty for a given measurement is one-fifth of the smallest increment on the device. When digital devices are used, uncertainties should be ± 1 of the smallest division that can be clearly read.

Invariably in the physics classroom, these measurements will need to be included as some part of a calculation. Proper procedure must be followed when applying mathematics to measurements with uncertainties². A summary of this procedure follows.

When performing addition or subtraction of measured values that include uncertainties, the resulting uncertainty is merely the sum of the individual *absolute* uncertainties. For example, suppose two measurements made with a meter stick and the difference between these is needed. Measurement A = $0.975 \pm 0.001 \text{ m}$ and B = $0.622 \pm 0.001 \text{ m}$. The resulting difference is

$$(0.975 \pm 0.001)\text{m} - (0.622 \pm 0.001)\text{m} = (0.353 \pm 0.002)\text{m}$$

Note that the resulting uncertainty is the *sum of* the individual uncertainties.

When multiplying or dividing measured values that include uncertainties, the resulting uncertainty is the sum of the individual *relative* uncertainties. Suppose an area is to be found using the above measurements, A and B. It is required first to change each absolute uncertainty into a relative uncertainty, as shown below.

$$\text{A: } \frac{0.001}{0.975} \cdot 100\% = 0.103\%$$

$$\text{B: } \frac{0.001}{0.622} \cdot 100\% = 0.161\%$$

Now the area can be found along with the uncertainty.

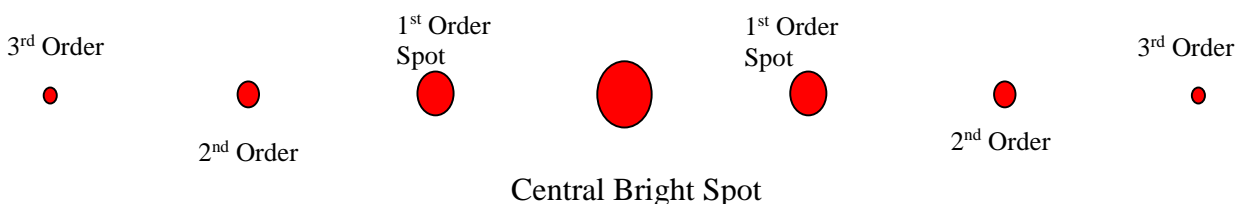
$$\text{Area} = (0.975 \pm 0.103\%)m \cdot (0.622 \pm 0.161\%)m = (0.606 \pm 0.264\%)m^2$$

This relative uncertainty may be converted to an absolute uncertainty, if desired. This is shown below.

$$0.00264 \cdot 0.606m^2 = 0.002m^2$$

So the resulting area may be expressed with a relative uncertainty as $0.606 \text{ m}^2 \pm 0.264\%$ or with an absolute uncertainty as $0.606 \pm 0.002 \text{ m}^2$.

To approach these concepts of measurement uncertainty and experimental error, this lab activity will use the measurement of a nanoscale phenomenon, the wavelength of laser light. Visible light has wavelengths from approximately 400 nm (violet) to 700 nm (red). The wavelength of visible light can be calculated by observing a diffraction pattern when this light passes through a diffraction grating³. A diffraction pattern is produced when a beam of light passes through small gaps and splits into, essentially, individual beams of light (Huygens's Principle). These individual beams of light interfere, both constructively and destructively. An example diffraction pattern is shown below⁴. The diffraction grating is used to create this pattern as it consists of numerous slits.



Sample diffraction pattern from a red laser through a diffraction grating.

As can be seen, the diffraction pattern is a series of bright spots projected on a screen or white board. The wavelength of the light can be determined by measuring the distance between the bright spots and the distance between the diffraction grating and screen and using trigonometry to determine the angular positions of the bright spots. By recording the uncertainty in each of their measurements, and propagating these uncertainties in their calculations, students will determine the wavelength of their laser with calculated uncertainty within a specific range; these are the error bars on their calculated data point. This value can then be compared to the manufacturer's acceptable range.

Variation in measurements that arise from other error sources may also be explored in this lab. Once all groups have determined a wavelength for each of the laser colors, the instructor will post these values and their uncertainties. All wavelength values obtained for a given color can

then be analyzed graphically and statistically. When plotted on a graph, do the measured wavelength values all lie within their respective uncertainty (i.e., their error bars), or do they scatter more widely than that? What is the mean of the measured wavelength values? To take this further, you may have students calculate the standard deviation and the standard error⁵. How does this compare with the manufacturer's specifications? Students can compare these values and discuss reasons for any variations, such as equipment drift or systematic measurement errors due to technique.

Sources:

1. Black, Dr. Timothy C., Physics 102 Lab 8: Measuring Wavelength with a Diffraction Grating, Spring, 2005.
2. <http://academia.hixie.ch/bath/laser/home.html>
3. <http://physics.appstate.edu/academicsprograms/undergraduate-programs/laboratory/measurement-and-uncertainty> Good, simple one for uncertainty measurements, no calculations.
4. http://physicshed.buffalostate.edu/pubs/MeasurementAnalysis/MA1_9ed.pdf More in-depth than #3. Handles calculations with uncertainties as well.
5. <http://www.radford.edu/~biol-web/stats/standarderrorcalc.pdf>

| Source/Website | Material |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|
| <p>Laser Classroom http://www.laserclassroom.com/laser-blox/multi-3-pack-free-curriculum-guide</p> | <ul style="list-style-type: none"> • one set of Laser Blox per group OR one laser per group |
| <p>Science Lab Supplies https://www.sciencelabsupplies.com/Diffraction-Grating-Slides-Double-Axis-13-500-Line-in.html</p> | <ul style="list-style-type: none"> • diffraction grating |

Materials (per group of 2 or 3):

- Lasers of different colors
- One diffraction grating with spacing indicated
- Meter stick
- Ring stands
- clamps

Advance Preparation:

Have all supplies set out ahead of time and organized. Place a complete set of lab supplies at each work station.

Safety Information:

Do not look directly into the laser.
 Do not shine the laser in the eyes of another person.

Suggested Instructional Procedure:

| Time | Activity | Goal |
|---------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Day 1&2 | Two days before the lab | |
| 90 – 100 min | Introduce students to the topic of measurement and uncertainty. Demonstrate how to accurately measure with a variety of devices, including uncertainties. Practice calculations with uncertainties Discuss why this important and difficult at the nanoscale. | To prepare students to measure correctly using different measuring tools. To prepare students to calculate uncertainties when performing mathematical operations, and to express their result as a range of acceptable values. |
| Day 2 | The day of the student lab | |
| 5 min | Students answer warm-up questions: -Why does error and uncertainty exist? -What level of precision is correct when using a meter stick? | To ensure students understand error, uncertainty, and how to measure correctly. |
| 40 - 45 min | Distribute <i>Student Worksheets</i> to students. Students follow procedures. | To allow students to observe a diffraction pattern, to measure points of constructive interference, and to calculate the wavelength of light along with the uncertainty. |
| 5 min | Clean up. | To prepare workspace for next class. |

Teaching Strategies:

1. Remind students to secure laser appropriately. If you are using a clamp to secure the laser, do not over tighten the clamp as it could damage the laser.
2. Secure the diffraction grating with a clamp so that it is perpendicular to the laser and so that the laser beam is directed through the center of the grating.
3. Students will need to adjust the distance between the laser, diffraction grating, and the screen (wall, whiteboard, etc.) until a clear and measurable diffraction pattern is seen.

Guided Dialog: Before beginning the lab, review the meaning of these terms:

Error - All measuring tools have some small but finite inaccuracy, that is, they are not absolutely accurate. “Error” in this sense is not a mistake, but occurs based on the precision at which a tool is made and the ability to read it accurately.

Uncertainty - A range of values in which a measurement is considered accurate.

Absolute uncertainty - The value of the uncertainty expressed in the same units as the measurement.

Relative uncertainty - The value of the uncertainty expressed as a percentage of the measurement.

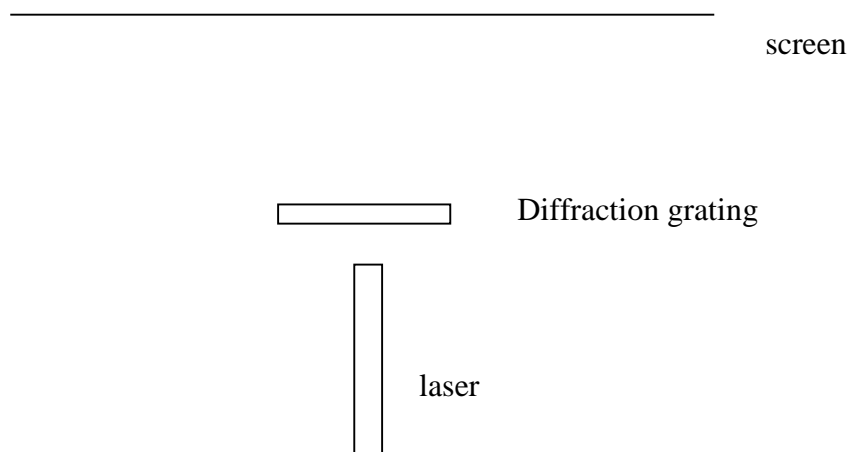
Wavelength - The distance between identical points on two successive waves.

Ask students questions to provoke thought and review what they already know:

1. To what level should measurements be made using a standard meter stick? *Values should be recorded to the half, fifth, or tenth of a millimeter. (Depends on which method is chosen by the instructor.)*
2. Review the procedures for handling uncertainties using different mathematical operations. *When adding or subtracting, absolute uncertainties are added together. When multiplying or dividing, relative uncertainties are added.*
3. Ask students if they think it is possible to use a meter stick to measure wavelengths of light at the nanoscale. *Explain that this will be the goal of this activity and they will determine if possible and with what accuracy.*

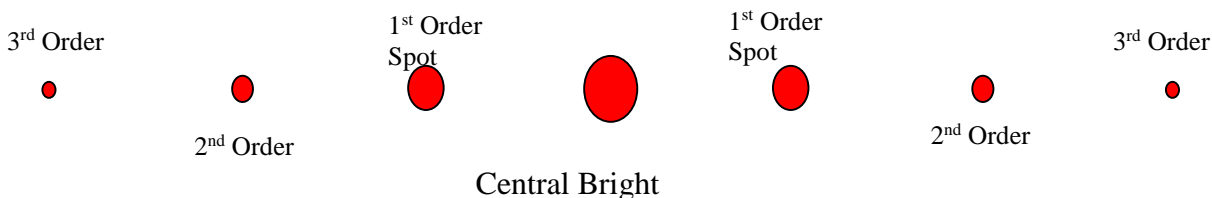
Procedure:

1. Set up your lab bench as shown in the diagram below. Ring stands not shown for clarity.



Overhead view of lab set-up

2. Be sure that laser is securely attached to its support and that the diffraction grating is perpendicular to the beam of the laser.
3. Record the grating spacing, in meters per line, using the information provided on the grating.
4. Turn the laser on and observe the diffraction pattern on the screen. If the pattern is seen vertically, rotate the diffraction grating 90 degrees. You should be able to see the central bright spot along with *at least* the 1st and 2nd order spots, as shown below. (Diagram not to scale.)

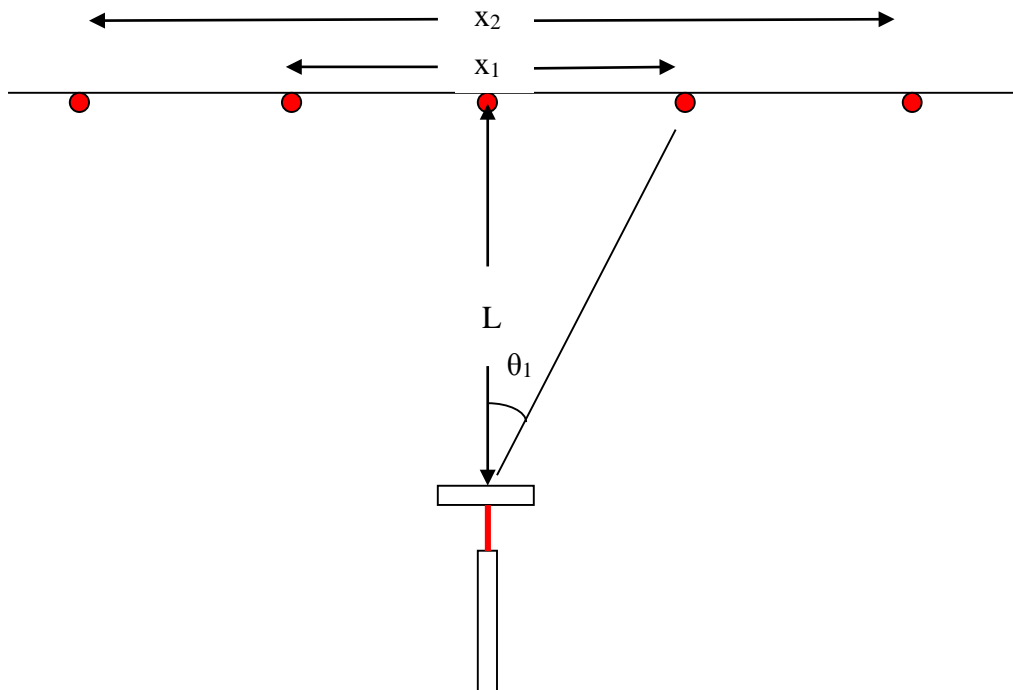


- Measure the distances between the two 1st order spots, the two 2nd order spots, and the two 3rd order spots, if possible. Record these in the table below. Be certain to include your uncertainty.
- Measure the distance between the diffraction grating and the screen. Record in the Data Table below. Include your uncertainty.

Data Table

| Color of Laser | d, Grating spacing, m/line | L, Distance from diffraction grating to screen, m | | x ₁ , Distance between 1 st order spots, m | | x ₂ , Distance between 2 nd order spots, m | | x ₃ , Distance between 3 rd order spots, m | |
|----------------|----------------------------|---------------------------------------------------|---------|------------------------------------------------------------------|---------|------------------------------------------------------------------|---------|------------------------------------------------------------------|---------|
| | | Value | Uncert. | Value | Uncert. | Value | Uncert. | Value | Uncert. |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

- Refer to the diagram below. Notice that θ_1 is the angle between the laser beam and the 1st order spot.



8. From trigonometry you can calculate the value of θ_1 , θ_2 , and θ_3 using the relationship

$$\tan \theta_m = \frac{x_m}{2L} \text{ where } m = 1, 2, 3, \dots$$

Record these values in the Results Table below. Be sure to include your uncertainty.

9. You can now use the relationship from optics to find the wavelength of your laser beam. Use the following relationship to solve for the wavelength, λ , in nanometers. Note that d is the grating spacing.

$$d \sin \theta_m = m\lambda \text{ where } m = 1, 2, 3$$

Record these values in the Results Table below. Be sure to include your uncertainty and express it as an absolute uncertainty.

10. Repeat for lasers of two different colors, or as instructed.

Results Table

| Color of Laser | θ_1 | | θ_2 | | θ_3 | | $\lambda_1, \text{ nm}$ | | $\lambda_2, \text{ nm}$ | | $\lambda_3, \text{ nm}$ | |
|----------------|------------|--------|------------|--------|------------|--------|-------------------------|--------|-------------------------|--------|-------------------------|--------|
| | Value | Uncert | Value | Uncert | Value | Uncert | Value | Uncert | Value | Uncert | Value | Uncert |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |

Cleanup: Return all supplies to their original locations.

Enhancing Understanding:

Use questions in the student worksheet to generate discussion.

Going Further: Students who have a good grasp of the content of the lab can be further challenged with these questions:

1. Considering the colors of the spectrum, at what wavelengths would you expect to find yellow, orange, and violet? *Answers may vary a little. Yellow is about 570 nm, orange is about 590 nm, and violet is about 400 nm.*
2. Measure the groove spacing (with uncertainty) in a CD or DVD. Compare results with acceptable values. *The approximate groove spacing for a CD is 1.6 μm and for a DVD it is 0.74 μm .*
3. In this lab, you demonstrated the ability to measure down to the ten-millionth of a meter, with typical classroom supplies. Research how light is used to measure small objects or what otherwise might prove difficult to measure. *Responses will vary. Examples may include: Interference patterns used to measure speed of light, the thickness of thin films is measured using light, expansion of the universe using the Doppler Effect, etc.*

Assessment:

Students should be able to:

1. Record a measurement with an acceptable uncertainty. Check that the precision of the uncertainty matches the precision of the measured value.
2. Perform calculations with measurements that include uncertainties. Check that uncertainty propagation was correct.
3. Explain how light is used to measure very small objects.

Resources: You may wish to use these resources either as background or as a resource for students to use in their inquiry-based design.

- Pohlman, Ken C., Digital Audio CD and Other Selected Digital Technologies Based on *Principles of Digital Audio, 4th Ed.*, (accessed July, 2013)
<http://mcgoodwin.net/digitalaudio/digitalaudio.html>
- The University of North Carolina at Chapel Hill, Department of Physics and Astronomy, "Introduction to Measurements and Error Analysis,"
<http://user.physics.unc.edu/~deardorf/uncertainty/UNCguide.html>
- Emery, Bob, "Measurement and Error,"
<http://webs.mn.catholic.edu.au/physics/emery/measurement.htm>
- National Institute of Standards and Technology <http://www.nist.gov/>
- Metrology: <http://en.wikipedia.org/wiki/Metrology>

National Science Education Standards (Grades 9 - 12)

Content Standard A: Science as Inquiry

- Abilities necessary to do scientific inquiry

Content Standard B: Physical Science

- Interactions of Energy and Matter

Content Standard E: Science and Technology

- Understandings about Science and Technology

Next Generation Science Standards (9-12)

HS-ETS1.A Defining and delimiting engineering problems

HS-ETS1.B Developing possible solutions

Student Worksheet (answers in red)

Uncertainty Measurements and the Wavelength of Light

Safety

Never look directly into a laser.
Never shine a laser into another person's eye.

Introduction

The closest star to the Earth, not counting our Sun, is about 26,000,000,000,000 miles away. The speed of light is about 300,000,000 meters per second. Mount Everest is about 29,029 feet high. The thickness of a human hair is about 0.0001 meters. An atom of gold has a radius of about 0.000000000135 meters or .135 nanometers.

All of these values are real and accepted by the scientific community. How are these values, both extremely large and extremely small, measured? Obviously, some of these values are approximate, while others are accurate. But *how* accurate are they?

In this lab, you will see firsthand how measurements account for accuracy and how measurements are reported to show this accuracy. You will also see that very small objects (in this case, the wavelength of light, at the nanoscale) can be measured fairly easily with typical science classroom supplies. When measuring on the nanoscale, accuracy is very important due to the extremely small size. Because of the small dimensions at the nanoscale it becomes imperative to be able to accurately measure materials and devices.

Light can be described as a wave, similar to that of ocean waves. In describing it as such, the distance between crests of successive waves (wavelength) can be measured. In fact, different colors of light have different wavelengths (and frequencies). This is what distinguishes between red from blue, or green, or violet, etc. As you will see, these wavelengths are very small on the nanoscale ~400-700nm. In this lab, you will measure the wavelength of three different colors of light and calculate the uncertainty of your measurement.

Materials

- Lasers of different colors
- Ring stands
- Diffraction Grating
- Screen (whiteboard, wall, etc.)
- Meter stick

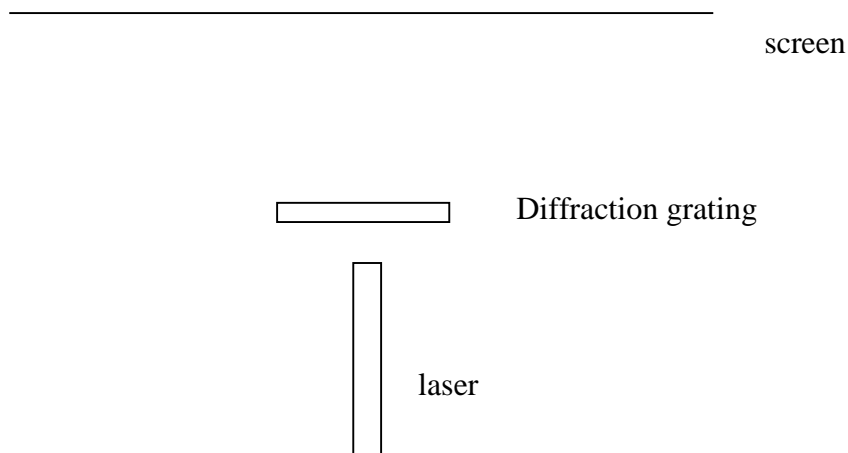
Question: Which color of light has the greatest wavelength? Using typical high school science lab supplies, within what percentage can you measure the wavelength of light?

Make a Prediction

Red has the greatest wavelength – 650nm. The percentage (relative uncertainty) will vary due to differing set-ups, but results should be within 5%.

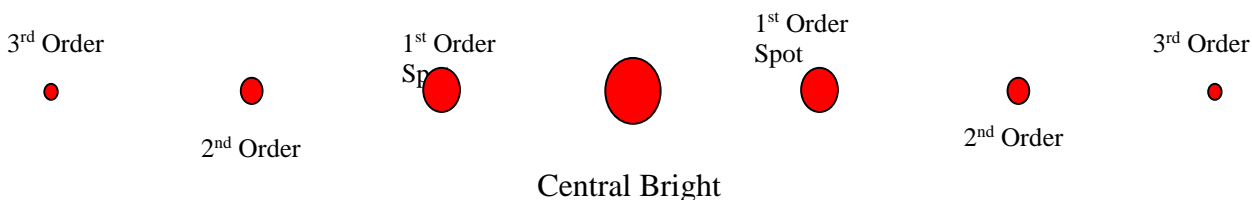
Procedure:

1. Set up your lab bench as shown in the diagram below. Ring stands not shown for clarity.

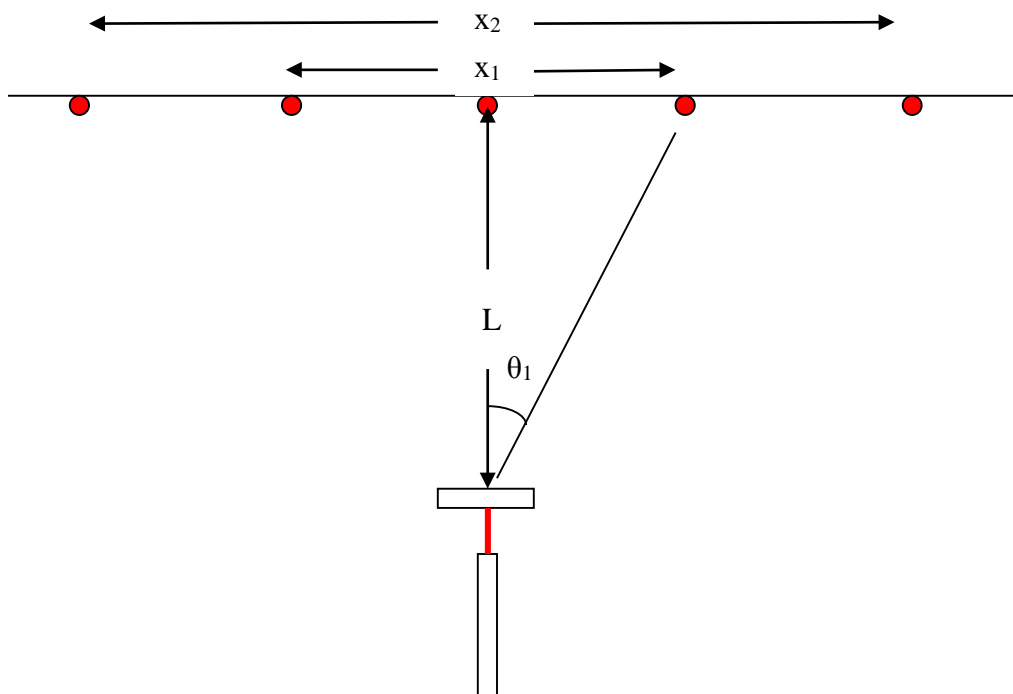


Overhead view of lab set-up

2. Be sure that laser is securely attached to its support and that the diffraction grating is perpendicular to the beam of the laser.
3. Record the grating spacing, in meters per line, using the information provided on the grating.
4. Turn the laser on and observe the diffraction pattern on the screen. If the pattern is seen vertically, rotate the diffraction grating 90 degrees. You should be able to see the central bright spot along with *at least* the 1st and 2nd order spots, as shown below. (Diagram not to scale.)



5. Measure the distances between the two 1st order spots, the two 2nd order spots, and the two 3rd order spots, if possible. Record these in the table below. Be certain to include your uncertainty.
6. Measure the distance between the diffraction grating and the screen. Record in the Data Table below. Include your uncertainty.
7. Refer to the diagram below. Notice that θ_1 is the angle between the laser beam and the 1st order spot.



8. From trigonometry you can calculate the value of θ_1 , θ_2 , and θ_3 using the relationship

$$\tan \theta_m = \frac{x_m}{2L} \text{ where } m = 1, 2, 3, \dots$$

Record these values in the Results Table below. Be sure to include your uncertainty.

9. You can now use the relationship from optics to find the wavelength of your laser beam. Use the following relationship to solve for the wavelength, λ , in nanometers. Note that d is the grating spacing.

$$d \sin \theta_m = m\lambda \text{ where } m = 1, 2, 3$$

Record these values in the Results Table below. Be sure to include your uncertainty and express it as an absolute uncertainty.

10. Repeat for lasers of two different colors, or as instructed.

Cleanup: Return all supplies to their original locations.

Data Table

| Color of Laser | d, Grating spacing, m/line | L, Distance from diffraction grating to screen, m | | x ₁ , Distance between 1 st order spots, m | | x ₂ , Distance between 2 nd order spots, m | | x ₃ , Distance between 3 rd order spots, m | |
|----------------|----------------------------|---------------------------------------------------|--------|------------------------------------------------------------------|--------|------------------------------------------------------------------|---------|------------------------------------------------------------------|---------|
| | | Value | Uncert | Value | Uncert | Value | Uncert. | Value | Uncert. |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

Analyze the Results:

Results Table

| Color of Laser | θ ₁ | | θ ₂ | | θ ₃ | | λ ₁ , nm | | λ ₂ , nm | | λ ₃ , nm | |
|----------------|----------------|--------|----------------|--------|----------------|--------|---------------------|--------|---------------------|--------|---------------------|--------|
| | Value | Uncert | Value | Uncert | Value | Uncert | Value | Uncert | Value | Uncert | Value | Uncert |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |

Clearly show your calculations here or on a separate sheet of paper:

Draw Conclusions:

1. What are the similarities and differences in the range of uncertainties? Why?

The range of uncertainties will only vary if the distances between the diffraction grating and the screen changes, for a given color. For a given set-up, the range of uncertainties will change due to changes in the spread of the diffraction pattern.

2. What are the similarities and differences in the values of wavelengths of light, for a given color? Should they be the same?

They should not be the same (between colors) but should be very similar for a given color between two different groups.

3. For a given color of laser, taking into account the range of uncertainties, do any groups' values of wavelength overlap? What does it mean when the overlap? What does it mean when they do not overlap?

Overlapping ranges of uncertainties indicate that two different groups can agree on a wavelength of light. If they do not overlap, these groups involved cannot agree on the wavelength of light for that color.

4. Create a graph of all the results for a given color of laser. Discuss these results.
-
-
-

5. How would the results change if students used a different measuring device other than a meter stick, or one with different gradations on it?

If the gradations are less precise, the range of uncertainties will increase.

6. Why do we need precise measurements at the nanoscale? When measuring such small dimensions even small measurement errors can have profound effects.
-
-
-